Master thesis

Modeling structural acoustic properties of the Beolab 9 loudspeaker

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Abstract

People knows that a loudspeaker radiates sound by vibration of the diaphragms of the speaker units. However the cabinet also radiate sound, which is not well studied.

In this project, a finite element model is used to simulate the cabinet of B&O loudspeaker Beolab 9 and illustrate the mechanical and structural acoustic properties. The effect of each components, the mode shapes of the cabinet, the mass-spring system, and the transmitting of vibrations are investigated in the simulations. Experimental results are also presented.

List of Figures

1	Beolab 9. $[1]$. 1
2	Inside view of the front cabinet.[1]	. 3
3	Cross stiffened plate and repeated section.	4
4	Linear orthotropic properties dialog in ANSYS.	9
5	Density for material dialog in ANSYS.	9
6	The ratio of natural frequencies of the simply supported square isotropic	
	plate obtained from analysis in MATLAB to that from simulation in	
	ANSYS	13
7	The ratios of the natural frequencies obtained from ANSYS simulation	
	to that from MATLAB. The natural frequencies are sorted pair by pair	
	of the same mode number.	16
8	The ratios of the natural frequencies of the equivalent plate obtained from	
	ANSYS simulation to that from MATLAB. Four different thicknesses of	
	the equivalent plate are studied.	18
9	Characterization of mode shape pattern of cylindrical shell and flat plate	
	[8]	. 19
10	SHELL93 geometry. [3]	20
11	SOLID92 geometry. [3]	21
12	COMBIN14 geometry. [3]	21
13	MASS21 geometry. [3]	22
14	The components of the B&O loudspeaker Beolab 9. [1]	23
15	Beolab 9 bottom and top dimensions. $[1]$	24
16	The geometry of the simple conical cabinet drew in SOLIDWORKS (left);	
	the meshing and bound conditions of the simple conical cabinet in $\ensuremath{\mathrm{ANSYS}}$	
	(right). \ldots	25
17	Operational deflecting shape of the cabinet at 257 Hz	. 27
18	Comparison of natural frequencies of modes of $m = 1$. n is taken from 1	
	to 5	. 27
19	Operational deflecting shape of the cabinet at 529 Hz	28
20	Operational deflecting shape of the cabinet at 520 Hz	29
21	The mode shape on the bottom of the cabinet simulation. \ldots \ldots \ldots	30
22	The cabinet with two holes for the mounting of the woofer unit and	
	midrange unit. Left part is the geometry made in SOLIDWORKS; right	
	part is the simulation in ANSYS	31
23	The cabinet with two holes for the mounting of the woofer unit and	
	midrange unit. The frame edges are attached on the hole flanges as thin	
	cylinders (steel rings).	31
24	Positions of the screws connecting front cabinet and rear cabinet. It is	
	simulated in ANSYS	32

25	Pins are simulated using rigid regions.	33
26	The effect areas of the screw.	34
27	The node mass element of the acoustic lens	34
28	The heat sink is made in ANSYS	35
29	The heat sink and the elastic cabinet consists a mass-spring system.	36
30	The position of stiffening anchor (red line) in the simulation of the Beolab 9.	37
31	Operational deflecting shape of the cabinet at 345 Hz in the simulation	
	of the Beolab 9 with all components attached	39
32	The circumferential nodal pattern of the cabinet (top view). Plus and	
	minus symbols indicate the phase of the deformed shape. Plus means	
	moving out and minus is moving in. The blue line in the middle shows	
	the position of the stiffening anchor. Six short lines show the position of	
	nodal lines.	39
33	The circumferential nodal pattern of mode $(m = 1, n = 2)$ from the	
	simulation without anchor, and the same mode half period later	40
34	The equipment settings of measuring accelerations of Beolab 9 cabinet.	42
35	Measure positions on the Beolab cabinet wall and bottom	43
36	Overlay of the measured accelerations of the Beolab 9 prototype I	44
37	The circumferential position is stretched into a straight line. \ldots \ldots \ldots	45
38	Normalized acceleration along the circumferential positions in the middle	
	of the cabinet of prototype III in a frequency range of 225 Hz to 240 Hz.	46
39	Normalized accelerations and phase of the circumferential positions of	
	prototype II at 57.5 Hz	47
40	Operational deflecting shape of the circumferential positions of prototype	
	II at 57.5 Hz	48
41	Normalized accelerations plotted along axial positions of left side (upper),	
	front (middle), and right side (lower) of prototype II cabinet at 57.5 Hz	48
42	Normalized accelerations plotted along the x and y axis of the bottom of	
	cabinet at 57.5 Hz	49
43	This is a inside view of the lower part of the front cabinet. The edges of	
	two stiffening ribs (green) are molded on the bottom	49
44	The operational deflecting shape (green line) of the simulation of Beolab	-
	9 prototype II at 156.5 Hz	50
45	Ribs between the bottom and the front cabinet, and the simulation in	F 1
10	ANSYS	51
40	Mode is annihilated by the anchor.	55
47	A new mode appears in the simulation of prototype III, since the anchor	
	changes the boundary condition of the cabinet wall. The dashed red line	F 0
40	shows the anchor.	56
48	Vibration transmits from the bottom to the cabinet wall	57

49	Operational deflections shapes of the bottom of prototype I from 0 Hz to 800 Hz
50	The ratios the natural frequencies of a equivalent plate obtained from ANSYS simulation divided by the natural frequencies from MATLAB 62
51	The ratio of natural frequencies of the simply supported square orthotropic plate obtained from analysis in MATLAB divided by natural frequencies obtained from simulation in ANSYS
52	The ratios of the natural frequencies obtained from MATLAB divided by the natural frequencies from ANSYS. $H' = 457$ Nm is taken the same as the one in ANSYS
53	Operational deflecting shape of the cabinet at 263 Hz
54	Operational deflecting shape of the cabinet at 257 Hz
55	Operational deflecting shape of the cabinet at 264 Hz
56	Operational deflecting shape of the cabinet at 289 Hz
57	Operational deflecting shape of the cabinet at 310 Hz
58	Operational deflecting shape of the cabinet at 348 Hz
59	Operational deflecting shape of the cabinet at 351 Hz
60	Operational deflecting shape of the cabinet at 473 Hz
61	Operational deflecting shape of the cabinet at 491 Hz
62	Operational deflecting shape of the cabinet at 496 Hz
63	Operational deflecting shape of the cabinet at 546 Hz
64	The operational deflecting shape of the simulation of Beolab 9 prototype
	I at 221 Hz
65	The operational deflecting shape of the simulation of Beolab 9 prototype
	I at 246 Hz
66	Overlay of the measured accelerations of the Beolab 9 prototype II $$ 70 $$
67	Overlay of the measured accelerations of the Beolab 9 prototype III 71
68	Normalized acceleration along the circumferential positions in the middle
	of the cabinet of prototype III in a frequency range of 0 Hz to 400 Hz 72
69	The operational deflecting shape plotted along the circumferential positions of the prototype II at 57.5 Hz
70	Operational deflecting shape of the left side (left) and right side (right) of the prototype II cabinet at 156.5 Hz 73
71	Modes $(m = 1, n = 2)$. The upper and middle parts show modes $(m = 1, n = 2)$ in the simulation of prototype II while the lower part shows
72	mode $(m = 1, n = 2)$ in the simulation of prototype III
14	changes the boundary condition of the cabinet wall 75
73	A mode $(m - 1, n - 2)$ of the midrange cabinet in a simulation of
10	n = 1, n = 2 or one incrange cabinet in a simulation of prototype I 76
	processive i

74	A mode of a mass-spring system consisted by the heat sink and the	
	cabinet in a simulation of prototype II. $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	77
75	Mode of a mass-spring system consisted by the heat sink and the cabinet	
	in a simulation of prototype III	78

List of Tables

1	Numerical factors for calculation of torsional rigidities. $[2]$	6
2	Dimensions of the cross-stiffened plate. The ribs are the same in the	
	x-direction and y-direction	8
3	Parameters used in ANSYS simulation for the equivalent (smeared) plate	
	of the stiffened plate	14
4	The first 16 natural frequencies and the corresponding mode numbers of	
	the analysis results and the ANSYS simulations of the equivalent plate. $% \mathcal{A}$.	15
5	Parameters used in ANSYS simulations for the equivalent plate of the	
	stiffened plate with four different thicknesses	17
6	Parameters of the first simulation of Beolab 9	25
7	Natural frequencies of modes (m, n) when m equals 1 and n is taken from	
	1 to 5	27
8	Natural frequencies of mode $(m = 1, n = 3)$ on the cabinet wall obtained	
	from each simulations.	37
9	Parameters used in ANSYS simulation of the equivalent plate of the bottom.	41
10	Comparison of natural frequencies and mode patterns of the prototype I	
	obtained from the simulation and the measurement. \ldots \ldots \ldots \ldots	52
11	Parameters of the new equivalent plate with the thickness of 8.2 mm	53
12	Natural frequencies and mode patterns in a simulation of the Beolab 9	
	prototype I	79
13	Natural frequencies and mode patterns obtained in the measurement data $% \left({{{\rm{D}}_{{\rm{B}}}}} \right)$	
	of prototype I	80
14	Natural frequencies and mode patterns in simulations of the Beolab 9	
	prototype II and III. \ldots	81

Contents

Li	st of	Figure	28	ii
Li	st of	Tables	3	vi
Co	onten	ıts		vii
In	trodu	uction		1
1	Equ	ivalent	plate of the stiffened plate	3
	1.1	Theori	es of the equivalent plate	3
	1.2	Simula	tion in ANSYS	7
		1.2.1	Parameters input in ANSYS	7
		1.2.2	Comparison of the analysis results and the results sim-	
			ulated in ANSYS	12
		1.2.3	Change the thickness of the equivalent model in ANSYS	16
2	FEN	A simu	lations of the Beolab9 cabinet	19
	2.1	Introd	uction of cylindrical modes	19
	2.2	FE-ele	ments used in the simulations	19
		2.2.1	FE-element SHELL93	20
		2.2.2	FE-element SOLID92	20
		2.2.3	FE-element COMBIN14	21
		2.2.4	FE-element MASS21	22
	2.3	Simula	tions for finding modes of the Beolab9 cabinet	23
		2.3.1	A simple model of the cabinet	24
		2.3.2	The equivalent properties	29
		2.3.3	A simple conical cabinet with two holes	30
		2.3.4	Two steel rings are attached to the hole flanges	30
		2.3.5	Joint	32
		2.3.6	Screws and pins	32
		2.3.7	Mass of the acoustic lens, midrange unit and woofer unit	33
		2.3.8	Heat sink	34
		2.3.9	Electronic part	35
		2.3.10	Stiffening anchor	36
		2.3.11	Natural frequencies comparison	36
		2.3.12	A simulation for the first prototype	40
3	Mea	asurem	ents on the Beolab 9 cabinet	42
	3.1	Equip	ment settings	42

		3.1.1 Measure positions on the cabinets	12
	3.2	Measurement data	13
		3.2.1 Measurement data of prototype I, II and III 4	13
		3.2.2 A study of the experimental data	15
4	Con	nparison and discussion 5	51
	4.1	Improve the simulation of the bottom	<i>j</i> 1
	4.2	Comparison of the simulation results and the measurement	
		data of prototype I	<i>5</i> 1
	4.3	Comparison of the simulation results and the measurement	
		data of prototype I	53
	4.4	Using ANSYS simulation as a tool	67
	4.5	The audible problem	58
Co	onclu	usions 6	60
Re	eferei	nces 6	51
\mathbf{A}	App	bendix 6	52
	A.1	Figures	32
	A.2	Tables 7	79

Introduction

The purpose of this project is to develop a mathematical/numerical model that can simulate the mechanical and structural acoustic properties of the B&O loudspeaker Beolab 9.

The Beolab9 is an active loudspeaker (that is a loudspeaker with a built-in amplifier) with a special geometry, see figure 1. Its bottom has a elliptical shape, and its body side is curved, which is illustrated in the right part of figure 1. A middle plate is used to split the cabinet into two chambers. A woofer, a midrange speaker driver, and an acoustic lens (for the tweeter) are mounted in the cabinet. The electronics and a large aluminum heat sink (cooling fin) are attached on the back of the cabinet.



Figure 1: Beolab 9. [1]

The first prototype of Beolab 9 had an audible problem caused by the vibration of the cabinet when reproducing sound. The audible problem was a peak around 140 Hz in the frequency response. The cabinet is made of foamed polystyrene with an average thickness of 6 mm. When the loudspeaker reproduces sound at low frequencies, the reaction force from the cabinet to the woofer acts on the cabinet, and therefore the cabinet is driven by the woofer.

In order to increase the stiffness of the cabinet, the second prototype used stiffening ribs on the inside surface of the cabinet wall. It was expected that the peak would be moved to a much higher frequency, say, higher than 1 kHz. Because at frequencies higher than 1 kHz, the signal to the woofer would be attenuated by a low pass filter, the driving force from the woofer would then be very small. Thereby, the audible problem would be too weak to be audible. Unfortunately, it was only increased to about 10 Hz, which is still in the woofer frequency range.

Next, an stiffening anchor was built in the cabinet from the left side to the right side of the cabinet. The peak disappeared from the frequency response. The reason will be discussed later.

To investigate the behavior of the prototypes of the Beolab 9, some simulations have been made, and the natural frequencies and mode shapes of the cabinet structure have been determined by the Finite Element program, ANSYS. The simulations take their starting point in a simple model, then the geometry is modified, and the components are built in one by one. An equivalent (smeared) shell/plate is used to simplify the model and to save calculation time.

The driving force from the woofer is simulated using sinusoidal pressure acting on woofer frame edge, which is attached on the cabinet. The acceleration of certain points on the cabinet are studied.

The loudspeaker prototypes are then driven by random noise from 0 Hz to 800 Hz and the acceleration of a number of positions on the cabinets are measured by B&K PULSE. Some important mode shapes are found in MATLAB by comparing the accelerations of each position at the natural frequencies.

The mode shape obtained from the measurement and from the simulations are compared and discussed.

1 Equivalent plate of the stiffened plate

The walls of the Beolab 9 cabinet are stiffened by ribs. Figure 2 shows an inside view of the front part of the cabinet. Crossed-stiffeners are attached on the cabinet shell. The plastic moulded cabinet shell is highly stiffened by integrated cross-stiffeners.



Figure 2: Inside view of the front cabinet.[1]

In order to simulate the stiffened shell/plate in ANSYS without requiring too much calculation time of the computer, equivalent properties will be introduced in this chapter.

1.1 Theories of the equivalent plate

The analysis of stiffened cabinet shell may be simplified by replacing the shell-stiffener combination by an equivalent homogeneous orthotropic shell of constant thickness. For simplicity the properties of a flat, stiffened plate will be considered in the following, where the bending stiffness, torsional rigidity and density of the equivalent plate will be introduced.

A simple stiffened plate is shown in the upper part of figure 3. The stiffeners are set to be the same in two perpendicular directions. The lower figure indicates the dimensions of a "repeated section" of the stiffened plate, where the dimensions h_{rib} and w are the height and width of the stiffener, h is the thickness of the plate, a_{rib} is the distance between stiffeners, and d is the distance from the plate bottom to the neutral axis (NA) of a "repeated section". The distance d can be determined from the following expression [2] (page 371),

$$d = \frac{(a_{rib} - w)h^2 + w(h_{rib} + h)^2}{2[(a_{rib} - w)h + w(h_{rib} + h)]}.$$
(1)



Figure 3: Cross stiffened plate and repeated section.

The bending stiffness per unit width in the x-direction, B'_x , can be found by taking into account the stiffeners in the x-direction only, as the stiffeners in the y-direction have nearly no effect on the bending stiffness in the x-direction. Therefore, it can be computed by the product of the Young's modulus, E, of the material and the area moment of inertia per unit width in the x-direction, I'_x .

$$B'_x = E \cdot I'_x,\tag{2}$$

where

$$I'_{x} = I'_{plate} + (d - h/2)^{2} \cdot h + I'_{rib} + (h_{rib} + h - d - h_{rib}/2)^{2} (w \cdot h_{rib})/a.$$
 (3)

In equation (3),

$$I'_{plate} = \frac{h^3}{12 \cdot (1 - \nu^2)} \tag{4}$$

is the local area moment of inertia per unit width of the plate, where ν is the Poisson's ratio of the plate; the second term of the right hand side is the shifting of the moment of inertia of the plate, as the NA has moved to the upper position after the stiffener attached on the plate;

$$I'_{rib} = \frac{w \cdot h_{rib}^3}{12 \cdot a_{rib}} \tag{5}$$

is the local area moment of inertia per unit width of the stiffeners, and the last term is the shifting of the moment of inertia of the stiffeners.

Since the stiffeners are the same in two directions, the equivalent plate might be an isotropic plate. The bending stiffness per unit width of this stiffened plate is therefore $B' = B'_y = B'_x$, where B'_x has been found from equations (2) and (3).

An approximate empirical formula of the torsional rigidity per unit width of the cross-stiffened plate had been found by Szilard [2] (page 77) as

$$H' = \frac{E \cdot h^3}{12(1-\nu^2)} + G \cdot \frac{h_{rib} \cdot w^3}{a} \cdot \eta, \tag{6}$$

where

$$G = \frac{E}{2(1+\nu)} \tag{7}$$

is the shear modulus of the plate, and η is a numerical factor depending on the ratio h_{rib}/w , which can be found in table 1.

h_{rib}/w	1.0	1.2	1.5	2.0	2.5	3.0	4.0	6.0	∞
η	0.140	0.166	0.196	0.229	0.249	0.263	0.281	0.299	0.333

Table 1: Numerical factors for calculation of torsional rigidities.[2]

The equivalent plate has a constant thickness, h_{eq} . As long as the considered wavelengths are larger than six times the thickness, the equivalent plate can be studied as a thin plate. [5] (page 53) Therefore, the natural frequencies can be calculated from the upper parameters. The equation of the natural frequencies of the simply supported orthotropic plate is [4] (page809)

$$f_{mn} = \frac{\pi}{2b^2} \sqrt{\frac{1}{m''}} \sqrt{B'_x(mb/a)^4 + 2H'n^2(mb/a)^2 + B'_y n^4},$$
(8)

where H' is the torsional rigidity of the orthotropic plate, a and b are the lengths of the plate edges in perpendicular directions. For the stiffened plate, H' is calculated form equation (6).¹ Note that the effect of the rotational inertia forces are neglected in the study.

In order to get the same mass as the stiffened plate, an equivalent density is calculated as

$$\rho = \frac{M''}{V''_{plate,eq}} = \frac{\rho_o(V''_{plate} + V''_{ribs})}{V''_{plate,eq}} = \frac{\rho_o(h + 2 \cdot h_{rib} \cdot w/a_{rib})}{h_{eq}}, \qquad (11)$$

where ρ_o is the density of the material, M" is the mass per unit area, V"_{plate} is the volume per unit area of the plate without ribs, V"_{ribs} is the volume per unit area of ribs in two perpendicular directions, V"_{plate,eq} is the volume per unit area of the equivalent plate, and h_{eq} is the thickness of the equivalent plate.

$$f_{mn} = \frac{1}{2\pi} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right] \sqrt{\frac{B'}{m''}},\tag{9}$$

where m, n are taken as $1, 2, 3, \dots, B'$ is the bending stiffness per unit width of the plate, and

$$m'' = \rho h \tag{10}$$

is the mass per unit area.

 $^{^1{\}rm The}$ natural frequencies of an isotropic plate simply supported along all edges is [6] (page 44)

The smeared equivalent plate is applicable at low and mid frequencies. As the frequency becomes higher, the wavelength in the plate becomes shorter. When a significant fraction (one quarter) of the wavelength becomes comparable with the distance between rib stiffeners, the equivalent model is no longer applicable.

The upper frequency limit of the equivalent plate is introduced,

$$c = \sqrt{\frac{E}{\rho}},\tag{12}$$

$$f = \frac{c}{\lambda} = \frac{c}{4a_{rib}},\tag{13}$$

where c is the wave propagation speed in the material, E is the Young's Modulus of the material, ρ is the density of the material, λ is the wave length, and a_{rib} is the distance between ribs.

The plate/shell in the following simulations has the material of Polystyrene, which has the Young's modulus of $2.1 \cdot 10^9$ N/m², and the density of 1030 kg/m³. The distance between stiffening ribs is 50 mm. The upper frequency limit is then obtained as 6.5 kHz by substituting the values into equation (12) and (13). The upper limit frequency, 6.5 kHz, is a relatively high frequency, as the frequency range of interest in the later simulations are mainly based on the frequency range of the woofer, which is less than 1 kHz.

1.2 Simulation in ANSYS

In this chapter, the behavior of a simply supported stiffened plate will be simulated in the Finite Element program, ANSYS, using the equivalent properties.

1.2.1 Parameters input in ANSYS

A simply supported square plate with stiffening ribs in the x-direction and ydirection has the following parameters. The Young's modulus of the material is $2.1 \cdot 10^{10}$ N/m², Poisson's ratio is 0.38 and dimensions of the stiffened plate are shown in table 2, where *a* is the length of the plate edge in the x-direction, *b* is the length of the plate edge in y-direction, a_{rib} is the distance between stiffening ribs, w is the width of the ribs, h is the constant thickness of the plate, and h_{rib} is the height of the ribs. The dimensions are similar with the cabinet wall of the Beolab 9, if the conical shape were opened and spread into a flat surface. Since the parameters are the same in the x-direction and y-direction, it is an isotropic plate.

Dimensions	a	b	a_{rib}	w	h	h_{rib}
[mm]	400	400	50	3.5	6	18.5

Table 2: Dimensions of the cross-stiffened plate. The ribs are the same in the x-direction and y-direction.

The Finite Element program, ANSYS, does not provide any functions for modeling the equivalent properties. In order to use the equivalent properties, the material parameters have to be recalculated for ANSYS. In other words, since ANSYS uses the standard formulas, the parameters obtained from chapter 1.1 must be calculated backwards to find the related basic parameters of the materials. In this way, ANSYS might be able to get the correct bending stiffness, torsional rigidity, and so on. The basic parameters are shown in figures 4 and 5. In the orthotropic properties dialog for a material in ANSYS, figure 4, EX and EY are the Young's modulus in the x-direction and y-direction; *PRXY*, *PRYZ*, AND *PRXZ* are Poisson's ratios of x-y plane, y-z plane, and x-z plane; GXY, GYZ, and GXZ are the Shear modulus in x-y plane, y-z plane, and x-z plane, separately. Note that the equivalent plate is in the x-y plane, and therefore the Shear modulus in y-z plane and x-z plane are not needed. The column of them should be empty. Similarly, the Young's modulus in the z-direction is for thick plate or solid structures. It should also be left empty. The density of the equivalent plate is different to the stiffened plate, because the stiffeners are smeared into the plate with a thickness of h_{eq} , and the mass of the stiffeners is added by changing the density of the plate. The density of the equivalent plate is calculated from equation (11), and it will be inputted in the Density for material dialog in ANSYS, see figure 5.

The procedure in the following is to calculate the material parameters of the stiffened plate first, and then substitute the parameters into the standard equations and find out the basic parameters.

The bending stiffness per unit width of the stiffened plate can be obtained

∧Linear 0	rthotropic	Propertie	s for	∎aterial.	🗵
Linear Orthotr	ropic Material	Properties :	Eor Mater	ial Number	1
Choose Poisso	n's Ratio				
	T1				
Temperatures					
EX		_			
EY	ļ				
EZ	ļ				
PRAI		-			
PRX7		-			
GXY		•			
GYZ	i				
GXZ					
Add Temper	rature Del	.ete Temperat	ure		Graph
		ОК	Cano	el l	Help

Figure 4: Linear orthotropic properties dialog in ANSYS.

\Lambda Density for Material Number 1 🛛 🗙
Density for Material Number 1
T1 Temperatures DENS
Add Temperature Delete Temperature Graph
OK Cancel Help

Figure 5: Density for material dialog in ANSYS.

in equations (2) and (3) as $B'_x = B'_y = 457$ Nm; while the torsional rigidity per unit width is calculated in equation (6) as H' = 48 Nm. Since the equivalent plate used in this chapter is an isotropic plate, B'_x will be used in the

following instead of B'_x and B'_y .

The parameters of the stiffened plate should be used for the equivalent plate. It looks like that the stiffened plate is smeared into a flat plate while keeping its mechanical behavior. In ANSYS, the standard formulas of B'_x and I'_{plate} are indicated in equations (14) and (15).²

$$B'_x = E \cdot I'_{plate}.$$
 (14)

$$I'_{plate} = \frac{h^3}{12 \cdot (1 - \nu^2)}.$$
(15)

The thickness of the equivalent plate is denoted as h_{eq} . Therefore h in equation (15) should be replaced by h_{eq} in the following calculation. It is set to 6 mm. ³ The related Young's modulus are obtained as

$$E_x = B'_x / I'_{plate},$$

$$E_y = B'_y / I'_{plate}.$$
(16)

With the given values, E_x and E_y are obtained to be $2.17 \cdot 10^{10} \text{ N/m}^2$. The Poisson's ratio is the same as the original value, 0.38, for all three directions. The Shear modulus in the x-y plane, G_{xy} , is calculated as ⁴

$$G = \frac{E_x \cdot E_y}{E_x + (1+\nu)E_y} \quad for \ orthotropic plate,$$

$$G = \frac{E}{2(1+\nu)} \quad for \ isotropic plate. \tag{17}$$

It is obtained that $G_{xy} = 7.9 \cdot 10^9 \text{ N/m^2}$. The standard formula of the torsional rigidity is given as [4] (page 510),

$$H' = \frac{1}{2} (\nu_y \cdot B'_x + \nu_x \cdot B'_y + 4 \cdot G_{xy} \cdot \frac{h^3}{12})$$

= $B'_x \cdot \nu + 2 \cdot G_{xy} \cdot \frac{h^3}{12}.^5$ (18)

 $^{^{2}}$ They are used in the calculations of FE-element SHELL93 [3]. The SHELL93 will be introduced in chapter 2.2. It is the element type used for the cabinet wall (shell) and bottom plate in Beolab 9 simulations.

³In chapter 1.2.3, different values of h_{eq} will be discussed.

 $^{{}^{4}}E$ can be also denoted as E_x for the isotropic plate.

The formula is simplified for an isotropic plate, and h equals h_{eq} , as the equivalent plate is been studing. However, this formula gives a higher value of H', 457 Nm, while the H' value of the stiffened plate is 48 Nm. It is almost ten times higher than the value that it is expected. H' cannot be changed directly in ANSYS, it is a hidden value, and is calculated from other parameters, B'_x , G, and h.

In order to adjust the value of H', either B'_x , G, or h has to be changed.

It is possible to adjust the value of H' by changing B'_x , however B'_x is also used in the calculation of natural frequencies in equation (8). If the value of H' is corrected in this way, the value of B'_x is lost. Therefore, the result would be wrong.

G can also be changed by adjusting the value of H'. It is written in [3] that the actual values of G_{xy} should be input; if not input, G_{xy} defaults to equation (17). That might be a chance to adjust the H' values by changing the G_{xy} value, while keeping B'_x and h_{eq} . In this way, the Shear modulus is not obtained from equation (17) but calculated to adjust the value of H'. From equation (18), the related Shear modulus G_{xy} is

$$G_{xy} = \frac{6}{h^3} (H' - \nu \cdot B'_x).$$
(19)

If H' value of the stiffened plate obtained in equation (6) is substituted into equation (19), G_{xy} will be obtained. Unfortunately, in this case the obtained G_{xy} is negative, which cannot be accepted by ANSYS. That means, the calculated G_{xy} cannot be used in ANSYS, and then the expected torsional rigidity would not show up. If a smaller G value is taken in the calculation, it is possible to get a more accurate value of H'. However, ANSYS has an error checking function, which forces the material to be real. ANSYS would stop the simulation before solving the model, if a fake material were detected. Therefore, the only choice is to use the standard formula of Shear modulus G.

The only chance left is to adjust the thickness of the equivalent plate. Since the thickness dominates the value of many parameters, such as I_{plate} and G_{xy} , we cannot just change the value of h_{eq} to adjust the value of H'. The relation between H' and h_{eq} is found by substituting equations (17), (15) and (16) into equation (18) as

$$H' = B'_{x} \cdot \nu + 2 \cdot G_{xy} \cdot \frac{h^{3}}{12}$$

$$= B'_{x} \cdot \nu + 2 \cdot \frac{E}{2(1+\nu)} \cdot \frac{h^{3}}{12}$$

$$= B'_{x} \cdot \nu + 2 \cdot \frac{B'_{x}}{I'_{plate} \cdot 2(1+\nu)} \cdot \frac{h^{3}}{12}$$

$$= B'_{x} \cdot \nu + 2 \cdot \frac{12 \cdot (1-\nu^{2}) \cdot B'_{x}}{h^{3} \cdot 2(1+\nu)} \cdot \frac{h^{3}}{12}$$

$$= B'_{x}.$$
(20)

Therefore, H' follows B'_x in the calculation. No matter what the value of h is, H' would not change. Note that it happens only because the related Young's modulus obtained from equation (16) is used to get the actual value of the bending stiffness. It never exists in the real case that the Young's of the material changes with the dimensions of the plate.

In conclusion, the effective torsional rigidity per unit width of the stiffened plate cannot be used in the equivalent plate in ANSYS simulation. The fact has to be noted that the simulation will be done with a high value of the torsional rigidity. How serious this affects the results will be discussed in chapter 1.2.2.

1.2.2 Comparison of the analysis results and the results simulated in ANSYS

In order to understand how ANSYS works on the plate, some simulations of the isotropic plate, orthotropic plate, and stiffened plates have been done in this chapter.

First, the natural frequencies of an isotropic plate are simulated in ANSYS and compared with the analysis results. This is to investigate whether the simulations in ANSYS can be used for the equivalent plate later by substituting the equivalent properties.

The plate in this simulation is a square plate, which has the size of 40 mm \times 40 mm \times 6 mm (a = b = 40 mm is the length of the edge, and h = 6 mm is the thickness). The plate is simply supported along all edges. It is made by Polystyrene, which has the Young's modulus of $2.1 \cdot 10^9$ N/m², Poisson's

ratio of 0.38, and a density of 1030 kg/m³.

The natural frequencies of the simply supported plate are calculated in MAT-LAB from equation (9).

The same model has been simulated in ANSYS as an isotropic plate. The meshing resolution is set to 1 of the Smart-mesh in ANSYS. The ratio of the natural frequencies of the analytical to simulated solutions are shown in figure 6. A good agreement between the analytical and simulated solutions is found below 2 kHz. The results can be improved at higher frequencies if finer meshing is taken in ANSYS. However, as mentioned before, the interesting frequency range of the later simulations are mainly the frequency range of the woofer, which is from 20 Hz to 1 kHz. Therefore, the upper limit 2 kHz is adequate.



Figure 6: The ratio of natural frequencies of the simply supported square isotropic plate obtained from analysis in MATLAB to that from simulation in ANSYS.

Similarly, in figure 51 in Appendix A.1, the ratio of the natural frequencies of analysis result to that of ANSYS model is made for an orthotropic plate, which has the edges, a and b, to 40 mm for both, and the thickness h to 6 mm. Its material is taken the same as the isotropic plate. The natural frequencies can be calculated from equation (8). It looks similar to figure 6, since the same geometry and material are used in the calculations.

Parameters	E_x	E_y	ν	G_{xy}	ρ	h_{eq}
Value	$2.17 \cdot 10^{10}$	$2.17 \cdot 10^{10}$	0.38	$7.6 \cdot 10^{8}$	1475	6
	$ m N/m^2$	$ m N/m^2$		$ m N/m^2$	$\rm kg/m^3$	mm

Table 3: Parameters used in ANSYS simulation for the equivalent (smeared) plate of the stiffened plate.

In figures 6 and Appendix A.1, figure 51, it can be further seen that the ratio has a slightly downward slope. It indicates that the density of natural frequencies obtained from ANSYS simulation is larger than the analysis results in MATLAB. The natural frequencies obtained in MATLAB are calculated in equations (8) and (9), where the effect of the rotational inertia forces are neglected. If the rotational inertia forces are included, the plate becomes stiffer, and therefore, the density of natural frequencies increases. It seems that ANSYS takes the effect of the rotational inertia forces into account, and a larger density of natural frequencies is found in ANSYS. The ANSYS simulation might be more accurate.

All in all, a good agreement below 2 kHz is found in figure 6 and also in figure 51 in Appendix A.1, proving that ANSYS works fine for finding the natural frequencies of the flat plate at low frequencies.

Next, the equivalent (smeared) plate of the stiffened plate is simulated in ANSYS.

The thickness of the equivalent plate is taken as the same thickness of the stiffened plate (without stiffeners). The parameters of the equivalent plate have been calculated from chapter 1.2.1, and shown in table 3.

In MATLAB, the natural frequencies of the equivalent plate is calculated from equation (8); while in ANSYS, a simulation is made to find the natural frequencies.

Comparing the natural frequencies obtained from MATLAB and from AN-SYS, an obvious difference is shown in Appendix A.1, figure 50. The data in the figure are the ratios of the natural frequencies obtained from MATLAB to that from ANSYS. The reason for the frequency deviation is that the effective torsional rigidity per unit width of the stiffened plate, H', is used for the equivalent plate in the MATLAB calculation; while it is not taken into account in ANSYS simulation. Instead, the value of H' in ANSYS follows

Natural frequencies [Hz]									
and the corresponding mode numbers									
Analysis	105	300	300	422	650	650	726	726	
Mode (m,n)	(1,1)	(1,2)	(2,1)	(2,2)	(1,3)	(3,1)	(2,3)	(3,2)	
ANSYS	140	351	351	559	701	701	908	908	
Mode (m,n)	(1,1)	(1,2)	(2,1)	(2,2)	(1,3)	(3,1)	(2,3)	(3,2)	
Analysis	949	1145	1145	1199	1199	1360	1360	1688	
Mode (m,n)	(3,3)	(1,4)	(4,1)	(2,4)	(4,2)	(3,4)	(4,3)	(4,4)	
ANSYS	1190	1190	1254	1395	1395	1738	1738	1815	
Mode (m,n)	(1,4)	(4,1)	(3,3)	(2,4)	(4,2)	(3,4)	(4,3)	(1,5)	

Table 4: The first 16 natural frequencies and the corresponding mode numbers of the analysis results and the ANSYS simulations of the equivalent plate.

the bending stiffness per unit width. It is mentioned in chapter 1.2.1 that the value of H' used in ANSYS is almost 10 times higher than the real value. It is clearly seen in equation (8) that the natural frequencies relate to the square root of the H' value. In other words, since the higher H' value is used in ANSYS, the natural frequencies obtained in ANSYS is higher than the analysis results obtained from MATLAB.

The natural frequencies are compared one by one from low frequencies to high frequencies. The first 16 natural frequencies are shown in table 4. It might be a problem if the frequency pair does not belong to the same mode number, as they are shown highlighted in the table. We have to compare the natural frequencies pair by pair of the same mode number, and then get figure 7. The ratios, which are close to one, correspond to mode numbers (1, x) and (x, 1) when x is greater than 2. Note that (1, x) overlaps (x, 1), as the plate is an isotropic plate. They are marked in figure 7 as (1, 3), (1, 4), and (1, 5). It can be noticed that the factor multiplied with H' in equation (8) is m^2n^2 . There are three terms in the square root. When m^2n^2 is much smaller than either m^4 or n^4 , the effect on the natural frequencies from the difference of H' is decreased. With the different values of m and n, the deviations of the natural frequencies are spread from 2% to 35% below 3 kHz. Actually, only the frequency range below 1 kHz is of interest. ⁶

In order to prove the reality of the upper comparison, a calculation is made

⁶In Beolab 9 simulations, the cabinet wall has a conical shape, and the mode is indicated with cylindrical patterns (m,n,s,r) instead of (m,n). The cylindrical patterns is introduced in chapter 2.1.



Figure 7: The ratios of the natural frequencies obtained from ANSYS simulation to that from MATLAB. The natural frequencies are sorted pair by pair of the same mode number.

in MATLAB using the same H' value as used in ANSYS to find the natural frequencies. It means that in equation (8), the value of H' is now 457 Nm instead of 48 Nm. The new natural frequencies are compared with the AN-SYS results, and a good agreement is shown in Appendix A.1, figure 52. It proves that the different values of H' used in the simulation in ANSYS are the problems of the natural frequency deviations.

1.2.3 Change the thickness of the equivalent model in ANSYS

The thickness of the equivalent plate was taken to 6 mm in the previous studies. It might be possible to use different thickness of the equivalent plate.

First, the density of the material is kept as what it is, and the relative thickness of the equivalent plate is calculated to get the same mass of the stiffened plate. If ρ is replaced by the material density, ρ_o , in equation (11), the thickness of the equivalent plate is then

$$h_{eq} = h + 2 \cdot h_{rib} \cdot w/a_{rib}. \tag{21}$$

Substituting the dimensions from table 2 in chapter 1.2.1 into equation (21), the thickness is obtained as 8.59 mm. Moreover, three more thickness are studied and shown together. 6 mm is the thickness studied in chapter 1.2.2;

Parameters	E_x	E_y	ν	G_{xy}	ρ
Units	N/m^2	N/m^2		N/m^2	kg/m^3
h = 6 mm	$2.17 \cdot 10^{10}$	$2.17 \cdot 10^{10}$	0.38	$7.6 \cdot 10^{8}$	1475
h = 8.59 mm	$7.42 \cdot 10^9$	$7.42 \cdot 10^9$	0.38	$2.68 \cdot 10^{9}$	1030
h = 12.25 mm	$2.55 \cdot 10^{9}$	$2.55 \cdot 10^{9}$	0.38	$9.26 \cdot 10^{8}$	722
h = 24.5 mm	$3.19 \cdot 10^{8}$	$3.19 \cdot 10^{8}$	0.38	$1.16 \cdot 10^{8}$	361

Table 5: Parameters used in ANSYS simulations for the equivalent plate of the stiffened plate with four different thicknesses.

12.25 mm is the middle thickness of the stiffened plate; 24.5 mm is the whole thickness of the stiffened plate. The related material parameters are calculated and shown in table 5.

Similar to figure 50 in Appendix A.1, the ratio of the natural frequencies of the equivalent plate obtained from ANSYS simulation to that from MAT-LAB is shown in figure 8. It is seen that the larger the thickness the smaller the ratio. At the frequency range below 1 kHz, the one with h = 24.5 mm shows the closest ratio to one, although, it goes downward very fast at higher frequencies. Because it is thicker, the effect of the rotational inertia forces are larger than others (the slope is steeper), as mentioned in chapter 1.2.2.

To make sure the plate is not too thick to be a "thin" plate. ⁷, the upper frequency limit is calculated as

$$f = \frac{c}{\lambda} = \frac{c}{6 \cdot h_{eq}},\tag{22}$$

where λ is the considered wavelength and c is the wave propagation speed in the plate, which can be obtained from equation (12). A value of the upper frequency limit is obtained as 6.4 kHz. Since the interesting frequency range is below 1 kHz, the limit is adequate.

All in all, it can be concluded that the larger the thickness of the equivalent plate the lower the natural frequencies. The better thickness of the equivalent plate is the sum of the thickness of the plate and the height of the rib. In the following simulations of Beolab9, both 6 mm and 24.5 mm will be used. Moreover, the thicknesses in between will be taken into account.

⁷When the considered wavelengths are larger than six times the thickness, the equivalent plate can be regarded as a thin plate [5] (page 53)



Figure 8: The ratios of the natural frequencies of the equivalent plate obtained from ANSYS simulation to that from MATLAB. Four different thicknesses of the equivalent plate are studied.

2 FEM simulations of the Beolab9 cabinet

Simulations of the Beolab 9 loudspeaker cabinet are made in a Finite Element program, ANSYS.

2.1 Introduction of cylindrical modes

In order to discuss the mode shapes of the cabinet, a characterization of mode shape pattern of cylindrical shell and flat plate is introduced in figure 9. For a cylinder m identifies the number of half-wavelengths in the axial direction and n is half of the number of nodal lines in the circumferential direction [7] (page 47); for a circular plate s is the number of nodal circles and r is the number of nodal diameters.



Figure 9: Characterization of mode shape pattern of cylindrical shell and flat plate [8].

Although the Beolab 9 cabinet has a conical shape, it is still similar to a cylinder. Therefore, the characterization of cylinder is used for the cabinet wall in the later discussion.

The characterization of the circular plate might be used for the Beolab 9 cabinet bottom, although the shape of the bottom is elliptic. The mode shape pattern of an elliptical plate is different from that of a circular plate because of the different length of the long axis and the short axis of the ellipse. However, it is easy to describe when we look into the mode shape in the following study.

2.2 FE-elements used in the simulations

Four FE-elements are used in the following simulations, SHELL93, SOLID92, COMBIN14, and MASS21.

2.2.1 FE-element SHELL93

The FE-element SHELL93 is used to simulate the Beolab 9 cabinet wall and plates.

SHELL93 is particularly well suited to model curved shells. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. The deformation shapes are quadratic in both in-plane directions. The element has plasticity, stress stiffening, large deflection, and large strain capabilities. [3] Figure 10 shows the geometry of SHELL93.



Figure 10: SHELL93 geometry. [3]

2.2.2 FE-element SOLID92

In order to simulate the woofer and midrange units frame edges, which are attached on the cabinet, a FE-element SOLID92 is chosen.

SOLID92 has a quadratic displacement behavior and is well suited to model irregular meshes. The element is defined by ten nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element also has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. The geometry, node locations, and the coordinate system for this element are shown in figure 11 [3].



Figure 11: SOLID92 geometry. [3]

2.2.3 FE-element COMBIN14

In order to suspend the Beolab 9 loudspeaker system, four springs are built on the bottom of the cabinet. The FE-element COMBIN14 is used to simulate the springs.



Figure 12: COMBIN14 geometry. [3]

COMBIN14 has longitudinal or torsional capability in 1-D, 2-D, or 3-D applications. The longitudinal spring-damper option is a uniaxial tensioncompression element with up to three degrees of freedom at each node: translations in the nodal x, y, and z directions. No bending or torsion is considered. The torsional spring-damper option is a purely rotational element with three degrees of freedom at each node: rotations about the nodal x, y, and z axes. No bending or axial loads are considered. The spring-damper element has no mass. The geometry, node locations, and the coordinate system for this element are shown in figure 12 [3].

The damping coefficient is set to 0 in following simulations, and therefore Beolab9 with springs is a purely spring-mass system. The spring constant is set to a very small value, and the natural frequencies of the spring-mass system is less than 1 Hz to avoid any effects on the cabinet in the interesting frequency range.

2.2.4 FE-element MASS21

The acoustic lens is mounted on top of the cabinet, and is simulated using a FE-element, MASS21, as a point mass.

MASS21 is a point element having up to six degrees of freedom: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes. A different mass and rotary inertia may be assigned to each coordinate direction. The coordinate system for this element is shown in figure 13 [3].



Figure 13: MASS21 geometry. [3]

2.3 Simulations for finding modes of the Beolab9 cabinet

Figure 14 shows the components of the B&O loudspeaker Beolab 9. From left to right, they are the heat sink (with electronics), the rear cabinet, the acoustic lens, the stiffening anchor, the foot ring, the front cabinet, the midrange unit, and the woofer unit. A horizontal plate between midrange and woofer divides the cabinet into two chambers. The material of the cabinet parts is foamed polystyrene, which has a density of 1030 kg/m³, and a Young's modulus of $2.1 \cdot 10^9$ N/m². The cabinet's overall general wall thickness is 6 mm, but locally this can be both larger and smaller. The cabinet wall is stiffened with ribs. Rib height inside cabinet is variable with approximate 16.7 mm in the middle of a cabinet part changing to approximate 20 mm at each side of the cabinet part - width varies from approximate 3 mm to 4 mm, due to draft and various directions [1]. It is a complex system. In order to study how the components affect the behavior of the loudspeaker, some simulations are made in this section.



Figure 14: The components of the B&O loudspeaker Beolab 9. [1]

Simulations of the B&O loudspeaker Beolab 9 is done step by step in AN-SYS to find the modes of the cabinet. The simulations start with simple model of the cabinet; the equivalent properties are taken into account; two holes are made in the cabinet to simulate the woofer and midrange unit's mounting hole; the flanges of those holes are stiffened by attaching the steel rings, which simulate the woofer and midrange units' frame edges; the cabinet is split into two pieces, front cabinet and rear cabinet, and the joint is simulated; the screws and pins connecting the front and rear cabinets are simulated; the mass of the acoustic lens, midrange unit, and woofer unit are added; the aluminum heat sink is attached on the rear cabinet; the mass of the electric part is added; in the end the stiffening anchor is mounted into the cabinet.

The procedure is carried out step by step, so that the effect on the cabinet from each modification or each component can be clearly identified.

2.3.1 A simple model of the cabinet

The starting model is a simple conical cabinet. The bottom and top of the model are ellipses, which have the same dimensions of Beolab 9, as shown in figure 15. The circle in the lower middle shows the position of the acoustic lens. A middle plate is used to split the cabinet into two chambers.



Figure 15: Beolab 9 bottom and top dimensions. [1]

The geometry of the model is drawn in SOLIDWORKS, as the curved side of the cabinet is very difficult to be drawn in ANSYS.⁸ An IGS file of the geometry is exported from SOLIDWORKS and imported into ANSYS. The left part of figure 16 shows the geometry made in SOLIDWORKS.

The model uses the same material of Beolab 9 cabinet; its overall wall thickness is 6 mm. The parameters used for the simulation are listed in table 6. The FE-element SHELL93 is used for the cabinet.

⁸The curved side is mentioned in figure 1



Figure 16: The geometry of the simple conical cabinet drew in SOLIDWORKS (left); the meshing and bound conditions of the simple conical cabinet in ANSYS (right).

Parameters	Density	Young's modulus	Poisson's ratio	Thickness
Values	$1030 \mathrm{~kg/m^2}$	$2.1\cdot 10^9~\mathrm{N/m^2}$	0.38	6 mm

Tab	le 6	j:	Parameters	of	${\rm the}$	first	simulation	of	Beolab	9.
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In order to suspend the whole cabinet, four pairs of massless-springs are attached on the bottom of the cabinet using FE-element, COMBIN14. The positions of the spring are shown in the right part of figure 16. Each spring pair consists of a vertical spring and a horizontal spring. One end of the vertical spring is fixed in x, y and z directions, the other end is connected to the bottom of the cabinet. The horizontal springs are connected in the same way, but perpendicular to the surface of the cabinet wall at the spring positions.

The mass-spring system has a fundamental frequency,

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k}{m}},\tag{23}$$

where k is the spring constant and m is the mass.

The fundamental frequency of the mass-spring system in the simulation is pretty low. The mass of the cabinet is about 4 kg. The spring constant is set as 200 N/m for each spring. Four vertical springs are in parallel, and therefore the sum of spring constant of vertical springs is 800 N/m. The fundamental frequency of the mass-spring system from equation (23) is less than 10 Hz. It is outside the interesting frequency range, 20 Hz to 1 kHz. The horizontal spring systems are more soft, since two springs are in series in each direction. That results in an even lower fundamental frequency.

When more and more components are added in the simulations, the mass becomes larger and larger. The fundamental frequency of the mass-spring system accordingly becomes smaller and smaller. Therefore, the springs can be used for all following simulations.

The first fifty modes of the model are found in ANSYS. The analysis type in ANSYS is chosen to be Modal, which means finding the natural frequencies of the simulated model. The results are the natural frequencies. However, some of the mode shapes are not clear, since their neighbor modes dominate the shape. It is better to call the shape as operational deflecting shape.

The first operational deflecting shape of the cabinet appears at 140 Hz, which is mode (s = 1, r = 0) on the bottom. ⁹ It is a pure mode without any dominating from other modes. The second one appearing at 257 Hz has operational deflecting shapes on the bottom and also on the cabinet wall. See figure 17. The upper left figure is a front view; the upper right figure is a right view; the lower left is an oblique view; while the lower right is a top view. ¹⁰ It is at the natural frequency of mode (m = 1, n = 3) of the cabinet wall.¹¹ The shape on the bottom is dominated by the bottom mode (s = 1, r = 1) at 263 Hz (see Appendix A.1, figure 53).

Table 7 shows a list of the natural frequencies of modes (m, n) of the cabinet wall when m equals 1 and n is taken from 1 to 5. They are plotted in figure 18. It is seen that a few higher mode show up at lower frequencies than the mode (1, 1). It is in line with the theory of a cylindrical shell ([7], Chapter 2). The mode shape figures can be found in Appendix A.1, from figure 54 to 63.

In table 7, all modes appear twice. Comparing appendix figures 54 and 55, we can see that the shapes look similar, however the shape rotates of an

⁹It is characterized as the mode shape patten of (s, r) shown in figure 9

 $^{^{10}}$ The model faces to the right in ANSYS

 $^{^{11}\}mathrm{It}$ is characterized as the mode shape patten of (m,n) shown in figure 9


Figure 17: Operational deflecting shape of the cabinet at 257 Hz.

Frequency [Hz]	257	264	289	310	348	351	472	491	496	546
Mode number n	3	3	2	2	4	4	1	5	5	1

Table 7: Natural frequencies of modes (m, n) when m equals 1 and n is taken from 1 to 5.



Figure 18: Comparison of natural frequencies of modes of m = 1. n is taken from 1 to 5.

angle, α . The value of the angle can be concluded from the figures that

$$\alpha = \frac{360}{2n} \cdot \frac{1}{2},\tag{24}$$

where 2n indicates the number of the nodal lines. The geometry is not a circular conical cylinder, but an elliptical one. The difference between the long axis and the short axis of the elliptical shape makes the frequency different.

Note that previous mentioned modes relates only to the woofer cabinet, which is the lower part of the cabinet between the bottom and the middle plate, and it is the wall of the woofer chamber. The middle plate, which departs the cabinet into two chambers, has strong effect on the cabinet wall. It generates a boundary condition of the cabinet wall between simply supported and clamped. Therefore, the cabinet wall does not behave as a whole model. The woofer cabinet and the midrange cabinet¹² should be studied separately. Looking at figure 19, it is clearly seen that the lower part has a mode shape of (2, 5) and the upper part is (1, 4).



Figure 19: Operational deflecting shape of the cabinet at 529 Hz.

 $^{^{12}{\}rm The}$ cabinet of the midrange chamber is the upper part of the cabinet higher than the middle plate.

At some condition, a mode shape does not behaves like the cylinder mode shape, because the shape of the cabinet is actually conical. The upper dimension is smaller than the lower dimension. Seen in figure 20, in the woofer cabinet, the mode shape is neither (2, 4) nor (2, 3). The lower circumferential mode pattern is n = 4 while the upper coincident to n = 3. Some patterns are not clear because of coinciding mode shapes.



Figure 20: Operational deflecting shape of the cabinet at 520 Hz.

The mode shapes on the bottom can be seen from the top view, when the cabinet wall is set to be transparent. The first six mode shapes on the bottom are shown in figure 21. Top left part of the figure is the mode (1,0) at 144 Hz; the top right is mode (1,1) at 256 Hz; the middle left is mode (1,1) at 313 Hz perpendicular to the previous mode (1,1); the middle right is mode (1,2) at 429 Hz; the bottom left is mode (2,2) at 464 Hz; and the bottom right is mode (1,2) at 578 Hz perpendicular to the one at 429 Hz.

2.3.2 The equivalent properties

The Beolab 9 has a stiffened cabinet wall. The equivalent properties of the stiffened plate is used to simulate the stiffened wall. Therefore, the model from last simulation can be reused by substituting the new parameters of the material. The parameters have been found in chapter 1.2.2, table 3.



Figure 21: The mode shape on the bottom of the cabinet simulation.

The stiffening ribs are only attached in the x-direction on the bottom and the middle plate. The stiffening ribs in x-direction do not stiffen the plate in the y-direction. Therefore, E_y uses the same Young's modulus of the material; while E_x for the bottom and middle plate is taken from 1.2.2, table 3.

2.3.3 A simple conical cabinet with two holes

In the cabinet of Beolab 9, the woofer unit and the midrange unit are mounted in the cabinet. Two holes should be made for the speaker units. As shown in the left part of figure 22, the modified geometry is implemented in SOLID-WORKS. After importing the geometry into ANSYS, a new simulation is done. The meshing and boundary conditions are shown in the right part of figure 22.

In the first simulation, the mode pattern order of the woofer cabinet was: (1,3), (1,3), (1,2), (1,2), (1,4), (1,4), (1,1), (1,5), (1,5), (1,1), and so on. After the holes are drilled in the cabinet wall, the mode pattern order becomes: (1,2), (1,2), (1,1), (1,1), (1,3), (1,3), (1,4), (1,4), and so on. In the following, the order keeps the same for the coming simulations until the heat sink is mounted on the cabinet.

2.3.4 Two steel rings are attached to the hole flanges

Frame edges of the woofer unit and the midrange unit is very stiff, not only because they are made by steel, but also because of the geometry of the frame. When the frame edges of the speaker units are mounted on the flanges of the cabinet hole, the cabinet wall is also stiffened. Therefore, it is necessary to



Figure 22: The cabinet with two holes for the mounting of the woofer unit and midrange unit. Left part is the geometry made in SOLIDWORKS; right part is the simulation in ANSYS.



Figure 23: The cabinet with two holes for the mounting of the woofer unit and midrange unit. The frame edges are attached on the hole flanges as thin cylinders (steel rings).

simulate the frame edges.

The holes flanges are extruded into thin cylinders with heights of 1 mm. See figure 23. FE-element SOLID92 are used for the cylinders to simulate the frame edges. The material is set to steel, and the Young's modulus is set to $2 \cdot 10^{12} \text{ N/m}^2$. The material is assumed to be ten times stiffer than a steel material to simulate the geometry stiffness of the frame.

2.3.5 Joint

In the new simulation, the cabinet is split into two parts, front cabinet and rear cabinet. The joint between the two parts is 1 mm thick, and the material of the joint is guessed to be 1000 times softer than the cabinet, since the joint of the Beolab 9 is made by an elastic glue. In chapter 4.3, a method to find the real parameters of the joint material will be introduced.

2.3.6 Screws and pins

Thirteen screws and four pins are used through the joint glue to mount the front cabinet and the rear cabinet. The pins are built on the rear cabinet and plug in the front cabinet.

Pink signs in figure 24 indicate the position of the screws. They are made by rigid region between node pairs in ANSYS. The displacement of the rigid regions in the x, y and z directions is fixed, but the rotations are free.



Figure 24: Positions of the screws connecting front cabinet and rear cabinet. It is simulated in ANSYS.

Four pins are also made by rigid regions, but with a displacement freedom in the x-direction. In the left figure of figure 25, red circles show the positions of the pins; the middle figure indicates the left corner where a screw and a pin are sitting; the right figure is a closer view of the screw and pin, and it shows that the upper one is the rigid region of the screw, while the lower is of the pin. The rigid region of the screw is connected between two nodes, one is sit on the front cabinet and the other on the rear cabinet. Both node are fixed in the x, y, and z directions. The rigid region of the pin is also connected between two node. The node on the rear cabinet is also fixed in the x, y, and z directions, but the one on the front cabinet is only fixed in y and z directions. It is free in the x-direction to simulate the moving of the pin in the front cabinet.



Figure 25: Pins are simulated using rigid regions.

The effect areas of the screws and pins can be spotted when we looks into the deformed shape. An example is shown in figure 26. The right figure shows only the lines of the left figure. Red arrows point at the position of the screw. Around that area the front and rear part of the cabinet are well connected. However, at the position a little further away from the screw, the connection is not as good any more. It is clearly seen in the right figure.

2.3.7 Mass of the acoustic lens, midrange unit and woofer unit

The mass of the woofer unit and midrange unit is added by increasing the density of the frame edges. The new density of the woofer frame edge is $1.03 \cdot 10^5 \text{ kg/m}^3$; while of the midrange is $1.58 \cdot 10^5 \text{ kg/m}^3$.



Figure 26: The effect areas of the screw.

The acoustic lens is simulated as a node element with a mass of 0.799 kg. The position is shown in figure 27 on top of the cabinet. The FE-element MASS21 is used for the simulation.



Figure 27: The node mass element of the acoustic lens.

2.3.8 Heat sink

The heat sink is attached to the rear cabinet. Twelve screws are used to mount the heat sink. They are simulated by rigid regions as treated for the screws between front and rear cabinets (see figure 28).

The heat sink is made of aluminum. It is very heavy and stiff. The large mass decreases the natural frequencies, and the stiffness increases some modal



Figure 28: The heat sink is made in ANSYS.

frequencies. Moreover, the large mass and the elastic cabinet (especially the joint) are a kind of mass-spring system. Figure 29 shows the deflecting shape of the mass-spring system. The dimensions of the deflection have been enlarged for clearly view.

In the previous simulations, the mode pattern order was found to be: (1, 2), (1, 2), (1, 1), (1, 1), (1, 3), (1, 3), (1, 4), (1, 4), and so on. After the heat sink is mounted on the cabinet, only one mode of (1, 3) appears, and the mode shapes become very complicated.

2.3.9 Electronic part

The electronic part in Beolab 9 is attached on the heat sink, because the heat sink is used to cool the inside amplifier. In the simulation, only the mass of the electronic part is taken into account. The mass is added into the density of the heat sink. The related density of the heat sink then becomes 3564 kg/m^3 .

After the mass of the electronic part is added on the heat sink, the mode (1,3) moved down to lower frequencies than the modes (1,1). Therefore, the new order of the mode patterns is: (1,2), (1,2), (1,3), (1,1), (1,1). The rest of the modes are too complicated to be figured out.



Figure 29: The heat sink and the elastic cabinet consists a mass-spring system.

2.3.10 Stiffening anchor

The stiffening anchor is mounted inside the cabinet. The material of the anchor is steel. It is simulated using rigid regions, since it is very stiff. The position of the stiffening anchor is shown in figure 30. The effect of the anchor will be illustrated in chapter 2.3.11.

After the anchor is mounted in the cabinet, one of the modes (1,2) disappears. The reason will be illustrated in chapter 2.3.11. The new mode pattern order is: (1,2), (1,3), (1,1), (1,1).

2.3.11 Natural frequencies comparison

The natural frequencies obtained from previous simulations are quite different, because of the adding steps of the components. They can only be com-



Figure 30: The position of stiffening anchor (red line) in the simulation of the Beolab 9.

pared with the same mode numbers. Table 8 shows the natural frequencies of mode (m = 1, n = 3) on the cabinet wall obtained from each simulation.¹³

Simple cabinet					
Simple cabinet with equivalent properties					
Holes made on the cabinet					
Woofer and midrange frame edges attached on the cabinet					
Cabinet is split into two parts and joint is made in between					
Screws and pins are added between front cabinet and rear cabinet					
Masses of woofer, midrange, and acoustic lens is added					
Heat sink attached on the rear cabinet					
Mass of electronic part added					
Stiffening anchor is mounted					

Table 8: Natural frequencies of mode (m = 1, n = 3) on the cabinet wall obtained from each simulations.

The natural frequency of the mode (m = 1, n = 3) on the cabinet wall obtained from the first simulation, a simple cabinet, is 257 Hz; when the

¹³There are two natural frequencies of mode (m = 1, n = 3) in simulations before the heat sink is taken into account. Only one of them, which has the same mode shape to the later simulations, is shown in table 8.

stiffening ribs are taken into account using the equivalent properties, the natural frequencies are moved to higher frequency, because the cabinet wall becomes stiffer; after two large holes are drilled in the cabinet, the natural frequency decreases, since the holes make the cabinet wall softer; the frame edges are very stiff, which increases the natural frequency; the soft joint decreases the natural frequency; Screws and pins increase the stiffness of the cabinet joint, and therefore push the natural frequency to a higher frequency; added mass of the speaker units reduces the natural frequency; the heat sink is stiff, which might increase the natural frequency. However, the large mass is a dominant factor that decreases the natural frequencies; the mass of the electronic part decreases the natural frequency more; the stiffening anchor does not have much influence on the natural frequency of this mode.

It is seen that previous simulations shows the effect of the components to the cabinet.

The anchor does not change the natural frequency of the mode (m = 1, n = 3). Figure 31 shows the mode shape of mode (m = 1, n = 3) in the last simulation. It can be found that the circumferential mode pattern is n = 3. There must be six nodal lines, but one of them is missing at the woofer position. One end of the stiffening anchor is placed on the left side of the cabinet; while the other end is on the right side. An indication figure 32 is made to show the top view clearly. The left side and right side moves in unti-phase, ¹⁴ and the distance between them are constant. Whenever the left and right sides of the cabinet move in unti-phase, the anchor does not have any effect on the mode. Actually, they are modes, which have odd number of n.

When n is even, the stiffening anchor works in a different condition. Figure 33 shows the circumferential nodal pattern of mode (m = 1, n = 2) from the simulation without anchor. The left part of the figure shows the mode shape at 248 Hz while the right part of the figure indicates the same mode but half period later. The left side of the cabinet moves in phase with the right side. When the left moves in, the right moves in, and vice versa. It it known from the mode pattern of mode (m = 1, n = 2) that when the left and right move in, the front and rear sides move out. Another mode of the same mode number can be found at 259 Hz. The nodal lines shift about 45 degrees. It also agrees with the calculation in equation (24) when n is 2. It can be noticed that the distance from the left side to the right side does

¹⁴One side moves out and the other side moves in.



Figure 31: Operational deflecting shape of the cabinet at 345 Hz in the simulation of the Beolab 9 with all components attached.



Figure 32: The circumferential nodal pattern of the cabinet (top view). Plus and minus symbols indicate the phase of the deformed shape. Plus means moving out and minus is moving in. The blue line in the middle shows the position of the stiffening anchor. Six short lines show the position of nodal lines.

not change in this mode shape.

In the simulation when the stiffening anchor is taken into account, only one



Figure 33: The circumferential nodal pattern of mode (m = 1, n = 2) from the simulation without anchor, and the same mode half period later.

mode is found at 265 Hz for mode number (m = 1, n = 2). It is similar to the latter mode in the previous paragraph. The mode that was shown in figure 33 does not appear in the simulation with the anchor. It could have been canceled by the anchor. More specifically, the mode shape could be shifted by the anchor with an angle, so that the anchor is sitting on the nodal lines. That angle, calculated from equation (24), is 45 degrees. Then the mode shape is coincident with another mode, which has the same mode number.

Therefore, the stiffening anchor annihilates a half of the even circumferential modes. More specifically, the anchor annihilates modes that have distance changing between two ends of the anchor. However, since the anchor changes the boundary conditions of the cabinet wall, new modes might appear at other frequencies. This will be proved in chapter 4.3.

2.3.12 A simulation for the first prototype

The parameters of the cabinet wall and the bottom are adjusted to simulate the first prototype. The first prototype of the Beolab 9 does not have the stiffening anchor and its cabinet wall is made without stiffening ribs.

To simulate the prototype, the cabinet wall is set to a shell with a constant thickness of 6 mm.

The parameters of the bottom are recalculated, since the stiffening ribs on the bottom are different of that on the cabinet wall. The bottom has stiffening ribs only in the x-direction. The thickness of the bottom plate (excluding ribs) is 6 mm; the thickness of the rib is 7 mm; the distance between ribs is 40 mm; the width of the rib is 3.5 mm. As mentioned in chapter 1.2.3, if the thickness of the equivalent plate is the sum of the plate and the rib, the

Parameters	E_x	E_y	ν	G_{xy}	ρ	h_{eq}
Value	$4.6 \cdot 10^{8}$	$2.1 \cdot 10^{8}$	0.38	$1.1 \cdot 10^{8}$	524	13
	N/m^2	N/m^2		N/m^2	$ m kg/m^3$	mm

Table 9: Parameters used in ANSYS simulation of the equivalent plate of the bottom.

result will be the best. Therefore, the thickness of the equivalent plate is set to the sum of the thickness of the bottom plate and the ribs, 13 mm. The parameters of the equivalent plate of bottom in table 9.

The natural frequencies and mode shape pattens are obtained. Some of the mode pattens are not clear, because at some frequencies when a number of modes coincide in a narrow frequency range, the mode shape is dominated by neighboring modes, and therefore it is not easy to figure out the mode pattern. The modes will be discussed in chapter 4.1, after the simulation has been adjusted with the measurement data, and then more simulations will be made for the other prototypes.

3 Measurements on the Beolab 9 cabinet

Three prototypes of Beolab 9 are examined experimentally in the laboratory. The prototype I is the first prototype, which has a cabinet wall without stiffening ribs; the prototype II has stiffening ribs that are used to stiffen the cabinet wall; the prototype III has an stiffening anchor mounted on prototype II. The bottom of all three prototypes are the same.



3.1 Equipment settings

Figure 34: The equipment settings of measuring accelerations of Beolab 9 cabinet.

The equipment setting is shown in figure 34. B&K PULSE is used to generate random noise in the frequency range of 0 Hz to 800 Hz. The PULSE output signal voltage level is 100 mV. The generator output drives the loudspeaker and is also used as a reference signal. The accelerometer is attached on the surface of the Beolab 9 cabinet. To avoid noise from the environment the frequency response between the acceleration and the reference signal is measured.

The accelerometer is calibrated by a standard calibrator.

3.1.1 Measure positions on the cabinets

Some positions are indicated on the cabinet wall and bottom in figure 35: twenty three positions spread on a horizontal circumference above the woofer on the cabinet wall;¹⁵ seven positions on the left side vertical line and another seven on the right side; five positions on the front vertical line; and sixteen position of the bottom.

All three Beolab 9 prototypes have the same measurement positions.



Figure 35: Measure positions on the Beolab cabinet wall and bottom.

In the following, when operational deflecting shapes of the woofer cabinet are plotted, only the lower part (between the bottom and the middle plate) of the vertical lines are taken into account, since the upper part of the cabinet (above the middle plate) belongs to the midrange cabinet.

3.2 Measurement data

Accelerations are measured at each position for all three Beolab 9 prototypes.

3.2.1 Measurement data of prototype I, II and III

The normalized accelerations of the measurement data of the Beolab 9 prototype I are plotted in figure 36; while the data of prototype II and III are plotted in Appendix A.1, figure 66, and figure 67. The upper part of the

 $^{^{15}\}mathrm{The}$ position of the stiffening anchor is 5 cm lower than the circumforerence.

figure is the overlay of the accelerations measured on the cabinet wall; while the lower part is on the bottom. At some frequencies, the accelerations are very high and clearly higher than the neighboring frequencies. They must be natural frequencies. However, at many frequencies the peak is not very clean. The operational deflecting shape is dominated by the neighboring modes or the vibrations on the bottom are dominated by the vibrations on the wall at some frequencies, and vice versa. Some of them are even masked by strong neighboring modes.



Figure 36: Overlay of the measured accelerations of the Beolab 9 prototype I .

In order to compare the accelerations, the largest value of the measured accelerations is found, and then all the data is normalized by the largest one.

3.2.2 A study of the experimental data

Some factors should be taken into account when we study the measurement data. As mentioned in chapter 2.3, the lower part of the cabinet (woofer cabinet) behaves separately from the upper part of the cabinet (midrange cabinet); the heat sink and the elastic cabinet constitute a mass-spring system; the operational deflecting shape might be dominated by neighboring modes; the conical shape of the cabinet results in different modes between the lower and higher parts of the woofer cabinet, and so on.

In order to find modes of the cabinet, the measured data are plotted in a 3D figure. The x-axis is the frequency; the y-axis is the position of the measurement points; the z axis is the normalized acceleration. The position of the measurement points is separated into the circumferential position in the middle of the cabinet, the left side, the front, the right side, the middle line in the x-direction on the bottom, and the middle line in the y-direction on the bottom. The circumferential position is stretched into a straight line, shown in figure 37.



Figure 37: The circumferential position is stretched into a straight line.

In the 3D figure, the influence from neighboring modes can be clearly seen. Appendix A.1, figure 68 is a 3D figure of the prototype III in the frequency range of 0 to 400 Hz. The frequency band is very wide. If we zoom in to a narrow frequency band, 225 to 240 Hz in figure 38, operational deflecting shapes are clearly seen. A solid green line indicates the operational deflecting shape at 228.5 Hz, and a dashed blue line is that at 231.5 Hz. It seems that

the solid green line has two peaks, one is around 20 cm in the circumferential positions and the other one is around 70 cm. However, the one around 20 cm is actually on the slope of the mode shape at a higher frequency. Therefore, the solid green line has only one peak, and it looks like the mode shape of mode (m = 1, n = 1). The dashed blue line is a little more complicated, because it lies on more slopes. But it could be noticed that the dashed blue line indicate a shape like mode (m = 1, n = 1), and this one is different from the previous one, because the peak shifts to another position. In other words, the shape rotates of an angle.



Figure 38: Normalized acceleration along the circumferential positions in the middle of the cabinet of prototype III in a frequency range of 225 Hz to 240 Hz.

Some shapes are really confusing if we look at the circumferential positions only. As shown in Appendix A.1, figure 69, the shape looks like a pure mode shape of mode (m = x, n = 1). ¹⁶ The lower part of of figure 39 shows the phase of the normalized accelerations. When the accelerations are positive, the phase is close to zero, and when the accelerations are negative, the phase is approximately $-\pi$. The operational deflecting shape is plotted in figure 40. However, if we look at the normalized accelerations plotted along axial lines on the left side, front, and right side, the mode is different. As seen in figure 41, the scale of the normalized acceleration of the positions on the front line of the cabinet is approximate ten times larger than that of the left and right side. Moreover, the front moves as a rigid, since a straight line is shown in the figure. It seems that the sides do not move much. More information is

 $^{{}^{16}}x$ can be set as 1, 2,

obtained in figure 42, the normalized accelerations of the positions on the bottom. The middle line of the bottom from the left side to the right side of the cabinet is in the x-direction; while that from the front to the back of the cabinet is in the y-direction. As shown in the lower part of the figure, the front of the cabinet is moving up and the rear part is moving down. All in all, the cabinet is rotating back and forth as a rigid body. It shows the behavior of a mass-spring system. The mass is the cabinet and the spring is the rubber foot ring.



Figure 39: Normalized accelerations and phase of the circumferential positions of prototype II at 57.5 Hz.

The bottoms of the three prototypes are almost the same. When the stiffening ribs are added on the cabinet walls of prototype II and III, the edges of two ribs are molded on the bottom, shown in figure 43. That changes the boundary conditions of the bottoms (along the edge of the bottom). In other words, the boundary conditions of the bottoms of prototype II and III are stiffer than that of prototype I. As mentioned before, the boundary condition is between simply supported and damped. The one of the prototype I is closer to the simply supported; while that of the prototype II and III is closer to the damped. Therefore, the natural frequencies of the bottom of prototype II and III are higher than prototype I. The natural frequencies of mode (s = 1, r = 0) are found to be 151 Hz, 156.5 Hz, and 166.5 Hz for prototype I, II and III, separately. The difference between the second one and the third one might be caused by the mass of the stiffening anchor and



Figure 40: Operational deflecting shape of the circumferential positions of prototype II at 57.5 Hz.



Figure 41: Normalized accelerations plotted along axial positions of left side (upper), front (middle), and right side (lower) of prototype II cabinet at 57.5 Hz.

perhaps production tolerances.

At the natural frequency of the bottom mode (s = 1, r = 0) of prototype II, an operational deflecting shapes of the woofer cabinet wall are noticed in figure 44 (green line). The shape looks similar to mode (m = x, n = 2).



Figure 42: Normalized accelerations plotted along the x and y axis of the bottom of cabinet at 57.5 Hz.



Figure 43: This is a inside view of the lower part of the front cabinet. The edges of two stiffening ribs (green) are molded on the bottom.

However, it is not likely that a mode is there. Two peaks can be clearly seen, because they are sitting on the slope of the neighboring modes; a dip appears in the middle, since it is on the slope of the neighboring dip. The real shape might be only one peak in the middle if the neighboring mode is not there. However, the shape would be hardly seen without the neighboring mode. The mass of the bottom is very small comparing with the mass of the cabinet wall and the components mounted on the cabinet wall. Therefore the effect the bottom to the cabinet wall is very weak. The shape will be illustrated later using ANSYS simulation in chapter 4.4.

Appendix A.1, figure 70 illustrates the operational deflecting shape of the



Figure 44: The operational deflecting shape (green line) of the simulation of Beolab 9 prototype II at 156.5 Hz.

left side (left) and right side (right) of the prototype II cabinet at 156.5 Hz. The green line shows the shape at 156.5 Hz. It is clear that the shape is not dominated by neighboring shapes, and the peaks are not in the middle of the sides but closer to the bottom. ¹⁷ It means that it is not an axial mode m = 1, but a mode driven by a force. Therefore, the operational deflection shape of the cabinet wall at the natural frequency of the bottom mode (s = 1, r = 0) might be dominated by the mode of the bottom. The discussion will be continued in chapter 4.4 when ANSYS simulation is used to find the shape.

From the previous mentioned method, some modes of prototype I are found and listed in Appendix A.2, table 13. 18

¹⁷The scale, Left side [cm], shows the height of the positions from the bottom to the middle plate of the prototype.

¹⁸The natural frequencies of mode (m = 1, n = 1) are highlighted, since the operational deflecting shapes are dominated by neighboring modes very much, and therefore they are difficult to be figured out.

4 Comparison and discussion

4.1 Improve the simulation of the bottom

Comparing with the measurement data, the simulations can be adjusted to get better results. The bottoms of three prototypes are almost the same. ¹⁹ The natural frequency of the mode (s = 1, r = 0) of the bottom of prototype I was 130 Hz in the simulation, but it is measured to 151 Hz. It was expected that the simulation result would be higher than the measurement data. Looking at the bottom of Beolab 9 prototype I in left part of figure 45, four ribs (colored to red in the figure) connect the bottom to the woofer hole flange in the front cabinet. They provide a strong connection between the bottom and the front cabinet. Therefore, four rigid regions are added in the simulation to simulate the connections, shown in the right part of figure 45. The natural frequency of the mode (s = 1, r = 0) of the bottom of the new simulation is 171 Hz. It looks like the expected value now. Moreover, the natural frequency of the mode (s = 1, r = 1) is 274 Hz, and the one of the measurement is 248.5 Hz. This is acceptable.



Figure 45: Ribs between the bottom and the front cabinet, and the simulation in ANSYS.

4.2 Comparison of the simulation results and the measurement data of prototype I

Natural frequencies and the corresponding mode patterns of the simulation of prototype I are listed in Appendix A.2, table 12. Only modes below 450

¹⁹Shown in chapter 3.2.2, figure 43, the only difference is that the edges of two stiffening ribs of the cabinet wall are molded on the bottoms of prototype II and III.

Comparison of modes of the prototype I							
Mode pattern	Natural fr	Difference					
	Simulation	Measurement					
(s=0, r=0)	171	151	13%				
(s=0, r=1)	274	248.5	10%				
(m = 1, n = 1)	257	269.5	-5%				
(m=1, n=1)	276	278	-1%				
(m=1, n=2)	122	129.5	-6%				
(m=1, n=2)	126	139	-9%				
(m = 1, n = 3)	197	147.5	34%				

Table 10: Comparison of natural frequencies and mode patterns of the prototype I obtained from the simulation and the measurement.

Hz are listed. Some of the modes are "Complicated", which means that the operational deflecting shape is dominated by many factors, so that it is too complicated to be figured out. The obtained natural frequencies of the simulation are compared with the measurement data of prototype I and listed in table 10. The mode pattern order is the same in the simulation and in the measurement, but the natural frequencies are not very close to each other.

The simulation model is very much simplified. The cabinet's overall general wall thickness of Beolab 9 prototype I is 6 mm, but at the area close to the joint between the front and rear cabinet, the thickness becomes 8 mm; while a constant thickness 6 mm is used for the cabinet wall in the simulation of prototype I. The material parameters of the joint is unknown, and is assumed to be 1000 times softer than the cabinet wall. The stiffness of the woofer and midrange frame edge is assumed to be ten times stiffer than a steel material to simulate the geometry stiffness. The details of the geometry, especially of the midrange cabinet and acoustic lens are not included.

Many factors can affect the simulated result. Therefore the simulation can not be expected to give very accurate natural frequencies. However, it can be used to show the behavior of the cabinet, such as mode shapes, mode orders, the influence of each components, and so on.

Parameters	E_x	ν	G_{xy}	ρ	h_{eq}
Value	$1.3 \cdot 10^{10}$	0.38	$4.6 \cdot 10^{9}$	1301	8.2
	$\rm N/m^2$		$ m N/m^2$	$ m kg/m^3$	mm

Table 11: Parameters of the new equivalent plate with the thickness of 8.2 mm.

4.3 Comparison of the simulation results and the measurement data of prototype I

The cabinet wall of prototype II and III has stiffening ribs. The distance between ribs has been adjusted and a better thickness of the equivalent plate is found.

The distance between the ribs was 50 mm. This was a coarse guess. Checking with the real cabinet wall, the value of the distance has been changed to 40 mm.

The thickness of the equivalent plate must be changed. Discussed in chapter 1.2.3, the thicker the equivalent plate the lower the natural frequencies, and a better value of the thickness of the equivalent plate was the sum of the thickness of the plate and the height of the rib, 24.5 mm. However, it was for the flat plate. When used it in the simulations of prototype II and III, the obtained natural frequencies were very low comparing with the measurement data. In the simulations of prototype II and III, the stiffened cabinet wall has a curved shape. It is unknown how the curvature affects the equivalent plate. Therefore, different thicknesses of the equivalent plate have been tried in order to determine an experimentally better value of the equivalent plate. 20 A better value is found to be one third of the sum of the thickness of the plate and the height of the rib, 8.2 mm. The corresponding parameters are listed in table 11.

The results of the simulations shows the behavior of the prototypes. The obtained natural frequencies of simulations of the prototype II and III below 500 Hz are listed in Appendix A.2, table 14. The natural frequencies of the same modes obtained from the two simulations are very close, within 5 Hz, except for four modes.

²⁰This is done by comparing the natural frequencies, 149 Hz, of mode (m = 1, n = 2), since this mode can be clearly seen in the measurement data. A risk exists here. In case, the mode at 149 Hz is not a mode (m = 1, n = 2), the natural frequencies of simulations of prototype II and III would be wrong, although the mode shapes are still usable and the comparison between simulations is valuable.

One of the four modes is (m = 1, n = 2). Shown in Appendix A.1, figure 71, the mode (m = 1, n = 2) appears twice in the simulation of prototype II and only once in the simulation of prototype III. As illustrated in chapter 2.3.11, the stiffening anchor annihilates half of the even circumferential modes. It rotates the mode shapes, so that the anchor is sitting on the nodal lines, and the rotated mode shape would be coincident with the other mode shape that has the same mode number. Therefore, the one at 149 Hz in the simulation of prototype II did not appear in the simulation of prototype III. The same thing happens in the measurement data. Shown in figure 46, the normalized accelerations of the circumferential positions of prototype II and III are plotted. A green line in the upper part of the figure shows the mode (m = 1, n = 2) of the prototype II at 149 Hz; while a green line in the lower part of the figure shows the operational deflecting shape at 149 Hz of the prototype III. It is almost a straight line. Therefore, the mode (m = 1, n = 2) disappears in prototype III.

Another one of the four is a new mode. Since the anchor changes the boundary conditions of the cabinet wall, a new mode appears at a higher frequency of 185 Hz. Shown in figure 47, the mode at 185 Hz has a nodal line at the anchor position, and the same mode cannot be found in the simulation of prototype II. A similar mode is found in the measurement data of prototype III at 168 Hz in Appendix A.1, figure 72. The left part of the figure shows the normalized acceleration on the circumferential positions. ²¹ A green line show the operational deflecting shape at 168 Hz. The shape looks like the mode shape of mode (m = x, n = 2). The right part of the figure shows the data of the left side positions. ²² It can be seen that the acceleration is close to zero at the anchor positions, 27 mm, and there is no neighboring modes dominating the mode shape. Therefore, it agrees with the simulated result.

The vibration of the cabinet at the natural frequency of the new mode is attenuated by the anchor, since the anchor divides the mode into two parts. It can be clearly seen in Appendix A.1, figure 72, that the upper part of the mode shape is very small; the lower part of the mode looks comparable with the canceled mode shown in figure 46, however, it is smaller than the canceled mode, since the length is smaller.

²¹The height of the circumferential positions are 5 cm higher than the anchor.

²²The data on the right side positions looks exactly the same as the left side, and therefore it is not shown to save space.



Normalized acceleration along the circumferential positions in the middle

Figure 46: Mode is annihilated by the anchor.

The other two of the four are modes of a mass-spring system consisted by the heat sink and the cabinet. The heat sink is the mass and the cabinet (especially the joint between the front cabinet and the rear cabinet) is the spring. Shown in Appendix A.1, figures 74 and 75, the heat sink vibrates up and down, and the joint is stretched. The operational deflecting shape of the woofer cabinet looks like a mode (m = 1, n = 2) in the simulation of prototype II at 229 Hz. The distance between the two ends of the anchor position is changing, since the anchor is not mount there. After the anchor is taken into account in the simulation of prototype III, the shape changes,



Figure 47: A new mode appears in the simulation of prototype III, since the anchor changes the boundary condition of the cabinet wall. The dashed red line shows the anchor.

and it looks like a shape of mode (m = 2, n = 2), but the upper is much smaller than the lower. The axial nodal line is at the position of the anchor. The natural frequency shifts slightly to 234 Hz.

The natural frequency of the mass-spring system is not accurate, since the material of the joint was guessed in chapter 2.3.5 to be 1000 times softer than polystyrene. If enough measurement positions are taken, the mode can be found in the measurement. Therefore, the real natural frequency can be used to adjust the Young's modulus of the joint. A simple way to do that is to try different values of the Young's modulus of the joint material, and obtain the natural frequencies, then pick up the closest one to the real natural frequency; meanwhile, the real value of Young's modulus is obtained. If this is done for one prototype, the obtained Young's modulus of the joint material can be used in the simulations of other prototypes.

4.4 Using ANSYS simulation as a tool

The simulations in ANSYS can be used as a tool to study behaviors of the cabinet.

At the end of chapter 3.2.2, it was found that the mode (s = 1, r = 0) of the bottom might dominate the operational deflecting shape on the cabinet wall. It is too tiny to be seen in the simulation, because the mass of the bottom is very light comparing with the mass of the cabinet (including the components). In order to investigate the vibration transmitting, the thickness of the bottom is increased to 16 mm instead of 6 mm.²³ A clear shape appears immediately. Seen in figure 48, the bottom moves down, the low part of the woofer edge follows the bottom, and therefore moves in. The woofer edge is supported by the cabinet wall. The middle horizontal line of the woofer edge works as an axis. When the lower part of the woofer edge moves in, the upper part moves out. Moreover, the vibration of the woofer edge influence the vibration of the cabinet wall.



Figure 48: Vibration transmits from the bottom to the cabinet wall.

²³The natural frequency will follow the changing of the thickness. However, the purpose is to show the operational deflecting shape. Therefore, the changing of the natural frequency does not matter.

Also, the vibration of the cabinet wall can be transmitted to the bottom. Since the mass of the cabinet wall and the components are much larger than the mass of the bottom, the operational deflection shapes of the bottom are really dominated by the modes on the cabinet wall. Seen from figure 49, it is really difficult to find the mode shape of the bottom because of the dominating of the mode on the cabinet wall, except mode (s = 1, r = 0), the deepest dip.



Figure 49: Operational deflections shapes of the bottom of prototype I from 0 Hz to 800 Hz.

At some frequencies, when modes on the midrange cabinet appear, the vibration might also be transmitted to the woofer cabinet. Shown in Appendix A.1, figure 73, the mode (m = 1, n = 2) of the midrange cabinet in the simulation of the prototype I appears at 157 Hz. The upper part of the heat sink vibrating in this mode, and therefore the vibration is transmitted into the lower part of the heat sink. Then the lower part of the heat sink drives the woofer cabinet. Unfortunately, there are not many measurement position on the midrange cabinet, so that modes of the midrange cabinet cannot been seen from the measurement data.

4.5 The audible problem

As mentioned in the introduction chapter, the prototype I has an audible problem around 140 Hz. It can be seen from the measurement data of proto type I that the highest peak of the normalized acceleration appears at 139 Hz. It is a mode (m = 1, n = 2). From the simulation of prototype I, a mode (m = 1, n = 2) appears at 126 Hz, which has the same mode shape of the measurement data.

When the stiffening ribs are attached on the cabinet wall in the prototype II, the natural frequency of mode (m = 1, n = 2) increases to 149 Hz, and that of the simulation is at 149 Hz as well. Since the mode shape is the same and the natural frequency does not change very much, the audible problem is still there.

After the stiffening anchor is mounted in the cabinet in prototype III, the mentioned mode is annihilated by the anchor. Although the anchor changes the boundary condition of the cabinet and a new mode appears, the new mode does not radiate much sound comparing with the canceled mode. As mentioned in chapter 4.3, the vibration of the cabinet at the natural frequency of the new mode is attenuated by the anchor. Therefore the audible problem has been solved.

Conclusions

The ANSYS model works fine on simulating the mechanical and structural acoustic properties of the B&O loudspeaker Beolab 9.

The simulations take their starting point in a simple model, and then the components are added one by one. This procedure helps us to understand how the components affect the cabinet and how the modes appear on the cabinet. Moreover, we can easily notice when an error happens somewhere and correct the model. If we did not do that, the result could go extremely wrong without showing anything, since the Beolab 9 is so complicated a system.

The simulations of the three prototype showed the mode shapes of the cabinet, mass-spring system, and the transmitting of the vibrations. The audible problem has been illustrated by showing the mode shapes. The effect of the stiffening anchor has been investigated.

ANSYS simulation can be used as a tool to investigate interesting phenomena. The vibration transmitted between the bottom and the cabinet wall has been illustrated using the simulations.

In future study, simulations can be improved in many ways. Mentioned in chapter 4.3, the parameters of material of the joint can be adjusted. Four ribs, which connects the bottom and the front cabinet, are simulated by rigid regions. Actually, they can be drawn in the geometry. The stiffening anchor is simulated by rigid region. It can also be made to a beam element in the simulation. The mass of the woofer unit is added on the edge of the frame. A large part of the mass is a magnet system of the unit. It is approximately 15 cm away from the frame edge. The moment of the mass should be computed by a product of the mass and the 15 mm. The simulation can be improved by adding a mass element at the right position. The midrange unit is similar to the woofer unit but smaller. The foot ring can also be simulated if it is drawn in the geometry.

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A Appendix

A.1 Figures



Figure 50: The ratios the natural frequencies of a equivalent plate obtained from ANSYS simulation divided by the natural frequencies from MATLAB.



Figure 51: The ratio of natural frequencies of the simply supported square orthotropic plate obtained from analysis in MATLAB divided by natural frequencies obtained from simulation in ANSYS.


Figure 52: The ratios of the natural frequencies obtained from MATLAB divided by the natural frequencies from ANSYS. H' = 457 Nm is taken the same as the one in ANSYS.



Figure 53: Operational deflecting shape of the cabinet at 263 Hz.



Figure 54: Operational deflecting shape of the cabinet at 257 Hz.



Figure 55: Operational deflecting shape of the cabinet at 264 Hz.



Figure 56: Operational deflecting shape of the cabinet at 289 Hz.



Figure 57: Operational deflecting shape of the cabinet at 310 Hz.



Figure 58: Operational deflecting shape of the cabinet at 348 Hz.



Figure 59: Operational deflecting shape of the cabinet at 351 Hz.



Figure 60: Operational deflecting shape of the cabinet at 473 Hz.



Figure 61: Operational deflecting shape of the cabinet at 491 Hz.



Figure 62: Operational deflecting shape of the cabinet at 496 Hz.



Figure 63: Operational deflecting shape of the cabinet at 546 Hz.



Figure 64: The operational deflecting shape of the simulation of Beolab 9 prototype I at 221 Hz.



Figure 65: The operational deflecting shape of the simulation of Beolab 9 prototype I at 246 Hz.



Figure 66: Overlay of the measured accelerations of the Beolab 9 prototype II .



Figure 67: Overlay of the measured accelerations of the Beolab 9 prototype III .



Figure 68: Normalized acceleration along the circumferential positions in the middle of the cabinet of prototype III in a frequency range of 0 Hz to 400 Hz.



Figure 69: The operational deflecting shape plotted along the circumferential positions of the prototype II at 57.5 Hz.



Figure 70: Operational deflecting shape of the left side (left) and right side (right) of the prototype II cabinet at 156.5 Hz.



Figure 71: Modes (m = 1, n = 2). The upper and middle parts show modes (m = 1, n = 2) in the simulation of prototype II while the lower part shows mode (m = 1, n = 2) in the simulation of prototype III.



Figure 72: New mode appears in the measurement of prototype III, since the anchor changes the boundary condition of the cabinet wall.



Figure 73: A mode (m = 1, n = 2) of the midrange cabinet in a simulation of prototype I.



Figure 74: A mode of a mass-spring system consisted by the heat sink and the cabinet in a simulation of prototype II.



Figure 75: Mode of a mass-spring system consisted by the heat sink and the cabinet in a simulation of prototype III.

A.2 Tables

Modes of a simulation of the Beolab 9 prototype I			
Natural frequency [Hz]	Mode pattern		
122	(m=1, n=2)		
126	(m = 1, n = 2)		
157	Mode of the midrange cabinet		
164	Mode of the midrange cabinet		
171	(s=1, r=0)		
187	Complicated		
197	(m=1, n=3)		
204	Mass - spring (Heat sink - cabinet)		
210	Mode of the midrange cabinet		
257	(m=1, n=1)		
274	(s = 1, r = 1)		
276	(m=1, n=1)		
283	Complicated		
292	Complicated		
297	Mode of the midrange cabinet		
311	Complicated		
342	Mode of the midrange cabinet		
352	Complicated		
375	Mode of the midrange cabinet		
380	Complicated		
413	Complicated		
425	Complicated		
430	(s=1, r=2)		
445	Complicated		

Table 12: Natural frequencies and mode patterns in a simulation of the Beolab 9 proto-type I.

Measurement data of the prototype I		
Mode pattern	Natural frequency [Hz]	
(s=1, r=0)	151	
(s=1, r=1)	248.5	
(m=1, n=1)	269.5	
(m=1, n=1)	278	
(m=1, n=2)	129.5	
(m=1, n=2)	139	
(m=1, n=3)	147.5	

Table 13: Natural frequencies and mode patterns obtained in the measurement data of prototype I.

Simulations of the Beolab 9 prototype II and III			
Mode pattern	Natural frequency [Hz]		
	Simulation II	Simulation III	
(m=1, n=2)	139	139	
(m=1, n=2)	149		
(s=1, r=0)	173	173	
(m=1, n=1)	180	180	
Forced by anchor		185	
(m=1, n=1)	192	194	
Complicated	208	213	
(m=1, n=3)	220	221	
Mass - spring (Heat sink - cabinet)	229		
Forced by anchor		234	
Mode of the midrange cabinet	239	244	
(s=1, r=1)	286	286	
Complicated	301	302	
Mode of the middle plate	335	335	
Complicated	344	345	
Complicated	357	359	
Complicated	372	375	
(m=2, n=3)	402	403	
Complicated	412	413	
(s=1, r=2)	437	437	
Complicated	450	451	
Mode of the middle plate	454	459	
(s=1, r=1)	491	492	

Table 14: Natural frequencies and mode patterns in simulations of the Beolab 9 prototype II and III.